A possibility of quark spin polarized phase in high density quark matter

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It is shown that the quark spin polarization may occur for each quark flavor by the use of the Nambu-Jona-Lasinio (NJL) model with a tensor-type four-point interaction between quarks, while the two-flavor color superconducting (2SC) phase in two-flavor case may be realized at high density quark matter.

§1. Introduction

One of recent interests in the physics governed by the quantum chromodynamics (QCD) may be to clarify the possibility of various phases under extreme conditions such as high temperature, high density, strong magnetic field and so on. Especially, in the region of high baryon density and low temperature, which phase exists is one of central problems, which is related to the understanding of inside of compact stars. For example, it is expected that the color superconducting phase is realized in quark matter. Here, another possibility with respect to the realized phase is investigated in the region of high baryon density and zero temperature in the context of the physics of compact stars. Namely, it is indicated that the quark spin polarization for each quark flavor may occur by the use of the NJL model with tensor-type four-point interaction between quarks.[1),2),3)]

§2. Two-flavor quark spin polarization versus two-flavor color superconductivity

Let us start with the following Lagrangian density:

$$\begin{aligned}
&\mathcal{L} = \bar{\psi} i\gamma^\mu \partial_\mu \psi + G_S((\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma_5 \vec{\tau}\psi)^2) + \mathcal{L}_T + \mathcal{L}_c \ , \\
&\mathcal{L}_T = -\frac{G}{4}\left((\bar{\psi}\gamma^\mu\gamma^\nu\vec{\tau}\psi)(\bar{\psi}\gamma_\mu\gamma_\nu\vec{\tau}\psi) + (\bar{\psi} i\gamma_5\gamma^\mu\gamma^\nu\psi)(\bar{\psi} i\gamma_5\gamma_\mu\gamma_\nu\psi)\right) \ , \\
&\mathcal{L}_c = \frac{G_c}{2}\sum_{A=2,5,7}\left((\bar{\psi} i\gamma_5\tau_2\lambda_A\psi^C)(\bar{\psi}^C i\gamma_5\tau_2\lambda_A\psi) + (\bar{\psi}\tau_2\lambda_A\psi^C)(\bar{\psi}^C\tau_2\lambda_A\psi)\right) \ . \quad (2.1)
\end{aligned}$$

Here, $\psi^C = C\bar{\psi}^T$ with $C = i\gamma^2\gamma^0$ being the charge conjugation operator. Also, τ_2 is the second component of the Pauli matrices representing the isospin $su(2)$-generator and λ_A are the antisymmetric Gell-Mann matrices representing the color $su(3)_c$-generator. Here, we adopt the mean field approximation in which operators $(\bar{\psi}\Gamma^A\psi)(\bar{\psi}\Gamma_A\psi)$ are replaced into $2\langle\bar{\psi}\Gamma^A\psi\rangle(\bar{\psi}\Gamma_A\psi) - \langle\bar{\psi}\Gamma^A\psi\rangle^2$ where $\langle\cdots\rangle$ represent the mean-field values. We concentrate on quark matter at high baryon density where the chiral symmetry is restored in the density region considered here, which leads to $\langle\bar{\psi}\psi\rangle = 0$. Further, we expand the quark field by using good helicity states. After that, we introduce the BCS state. As a result, we can derive the thermodynamic potential Φ as follows:

$$
\begin{aligned}
\Phi(\Delta, F, \mu) &= 2\cdot\frac{1}{V}\sum_{\boldsymbol{p}\eta(\varepsilon_{\boldsymbol{p}}^{(\eta)}\leq\mu)}\left[2(\varepsilon_{\boldsymbol{p}}^{(\eta)}-\mu)-\sqrt{(\varepsilon_{\boldsymbol{p}}^{(\eta)}-\mu)^2+3\Delta^2 f_p(\eta)^2}\right] \\
&\quad +2\cdot\frac{1}{V}\sum_{\boldsymbol{p}\eta(\varepsilon_{\boldsymbol{p}}^{(\eta)}>\mu)}^{\Lambda}\left[(\varepsilon_{\boldsymbol{p}}^{(\eta)}-\mu)-\sqrt{(\varepsilon_{\boldsymbol{p}}^{(\eta)}-\mu)^2+3\Delta^2 f_p(\eta)^2}\right] \\
&\quad +\frac{F^2}{2G}+\frac{3\Delta^2}{2G_c}\ , \qquad (2.2) \\
F &= -G\langle\bar{\psi}\Sigma_3\tau_3\psi\rangle, \quad \Sigma_3 = -i\gamma^1\gamma^2 = \begin{pmatrix}\sigma_3 & 0\\ 0 & \sigma_3\end{pmatrix}, \\
\Delta_A &= \Delta_A^* = -G_c\langle\bar{\psi}^C i\gamma_5\tau_2\lambda_A\psi\rangle, \\
f_p(\eta) &= \frac{p+\eta e}{\varepsilon_{\boldsymbol{p}}^{(\eta)}}\ , \qquad p=\sqrt{p_1^2+p_2^2+p_3^2}\ , \qquad e = F\frac{\sqrt{p_1^2+p_2^2}}{p}\ ,
\end{aligned}
$$

where $\varepsilon_{\boldsymbol{p}}^{(\eta)} = \sqrt{p_3^2+\left(F+\eta\sqrt{p_1^2+p_2^2}\right)^2}$ represents the single quark energy, and μ, $\eta=\pm$ and V represent the chemical potential, the helicity and the volume, respectively. Here, F and $\Delta = \Delta_2 = \Delta_5 = \Delta_7$ represent the quark spin polarized condensate (spin alignment) and the color superconducting gap, respectively.[*)] Figure 1 shows the contour map of the thermodynamic potential. The parameters used here are $G = 20.0$ GeV, $G_c = 6.6$ GeV and the three-momentum cutoff $\Lambda = 0.631$ GeV, respectively. In the region with small chemical potential such as $\mu = 0.40$ GeV, the thermodynamic potential has the minimum point with $F = 0$ and $\Delta \neq 0$, which is nothing but the 2SC phase. As the chemical potential is increasing, the point with $F \neq 0$ and $\Delta \neq 0$ gives a minimum point. Finally, in the region with large chemical potential such as $\mu = 0.50$ GeV, the Thermodynamic potential reveals minimum with $F \neq 0$ and $\Delta = 0$, which is identical with the spin polarized phase. Thus, the phase transition from 2SC to SP phases occurs.

[*)] In,[2)] an approximation that $f_p(\eta) = 1$ is adopted. Here, we retain the factor $f_p(\eta)$. The qualitative behavior of the thermodynamic potential is not changed, but quantitatively, the thermodynamic potential is slightly modified and the behavior of the thermodynamic potential becomes smoother.

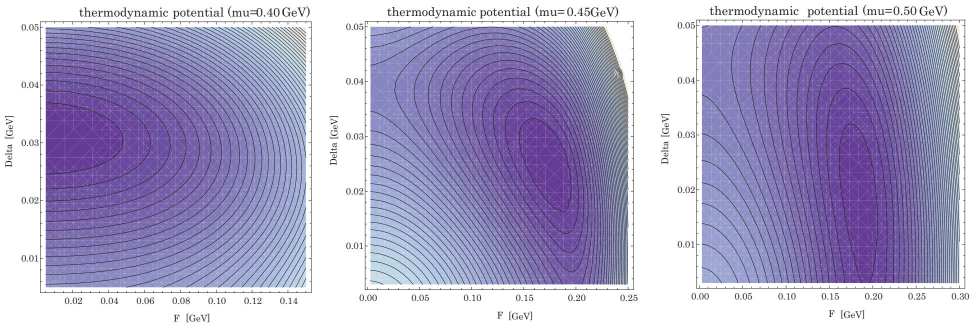

Fig. 1. The contour map of thermodynamic potential of each value of chemical potential μ is shown. The vertical and horizontal axis represent the color superconducting gap $\varDelta$ and the spin polarized condensate F, respectively.

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References

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